

Pennsylvania Content Standards Grade Level Benchmarks

2.1. Numbers, Number Systems and Number Relationships

9	10	11	12
<p>A. Describe and use inverse operations to solve linear equations, inequalities and absolute value.</p> <ul style="list-style-type: none"> • Use number line graphs to represent integers, absolute value, powers, and roots. • Describe number line graphs represented by absolute value. • Simplify and estimate roots. 	<p>A. Describe and apply inverse operations (reciprocal, absolute value, raising to power, finding roots and logarithms) to solve equations and inequalities.</p> <ul style="list-style-type: none"> • Use absolute value to represent distance. • Apply roots to solve Pythagorean theorem applications. 	<p>A. Use operations such as opposite, reciprocal, absolute value, raising to a power, finding roots and logarithms.</p>	<p>A. Use operations such as opposite, reciprocal, absolute value, raising to a power, finding roots and logarithms.</p> <ul style="list-style-type: none"> • Order and describe verbally, graphically, and using set notation subsets of real numbers. • Describe domain and range of functions modeled from real life data. • Represent and analyze finite graphs as relations, matrices, and pictorial representations. • Define, represent, graph and apply complex numbers to real world situation.

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2.2. Computation and Estimation

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<p>A. Describe and apply computation concepts, operations, and procedures involving real numbers.</p>	<p>A. Develop and use computation concepts, operations and procedures on real numbers in problem solving situations.</p>	<p>A. Develop and use computation concepts, operations and procedures on real numbers in problem solving situations.</p>	<p>A. Determine and explain the meaning of the zeros of functions model from real life situations.</p>

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<ul style="list-style-type: none"> • Simplify irrational expressions involving square roots. • Perform calculations by applying rule for exponents and roots. • Define fractional exponents and rewrite in radical form. <p>B. Apply scientific notation to estimate very large and small numbers.</p> <p>C. Construct and apply mathematical models, including lines and curves of best fit, to estimate values of related quantities.</p> <p>D. Recognize, identify, and correct computational errors.</p> <p>E. Interpret tolerance statements (e.g. $5 \pm .001$ units).</p>	<p>B. Use estimation to solve problems for which an exact answer is not needed.</p> <ul style="list-style-type: none"> • Estimate areas, lengths, and volumes of geometric figures. <p>C. Construct and apply mathematical models, including lines and curves of best fit, to estimate values of related quantities.</p> <p>D. Describe and explain the amount of error that may exist in a computation using estimates.</p> <ul style="list-style-type: none"> • Estimate and justify reasonableness of calculations. • Express estimates using appropriate units of measure. <p>E. Use appropriate instruments to measure and analyze the appropriate degree of precision.</p>	<p>B. Use estimation to solve problems for which an exact answer is not needed.</p> <p>C. Construct and apply mathematical models, including lines and curves of best fit, to estimate values of related quantities.</p> <p>D. Describe and explain the amount of error that may exist in a computation using estimates.</p> <p>E. Recognize that the degree of precision needed in calculating a number depends on how the results will be used and the instruments used to generate the measure.</p>	<ul style="list-style-type: none"> • Develop and use computation, operations and procedures on real and complex numbers in problem solving situations <p>B. Apply appropriate use of estimation and degree of precision in problem solving.</p> <p>C. Construct and apply graphs of polynomial functions modeled from real data.</p>

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<p>F. Demonstrate skills for using computer spreadsheets and scientific and graphing calculators.</p>	<p>F. Demonstrate skills for using computer spreadsheets and scientific and graphing calculators.</p>	<p>F. Demonstrate skills for using computer spreadsheets and scientific and graphing calculators.</p>	<p>F. Use graphing utilities to determine best fit functions (e.g. linear, quadratic, cubic, exponential, power, etc.)</p>

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2.3. Measurement and Estimation

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<p>A. Demonstrate measurement techniques using appropriate tools (e.g. ruler, protractor, micrometer, caliper.)</p> <p>B. Describe the relationship among linear dimensions, area, and volume and the corresponding uses of units, square units, and cubic units of measure in the real world.</p> <p>D. Describe and give examples of the need for precision and significant digits as they relate to measurement and how units indicate precision.</p>	<p>A. Explain appropriate degree of accuracy required to solve real world problems that involve the concepts of absolute and relative errors in measurements.</p> <p>B. Define radians and convert between radians and degrees</p> <p>C. Represent the relationship among linear dimension, area, and volume and the corresponding uses of units, square units, and cubic units of measure using physical models.</p> <p>D. Create and write and solve real world problems that involve the concepts of absolute and relative errors in measurements.</p>	<p>A. Select and use appropriate units and tools to measure to the degree of accuracy required in particular measurement situations.</p> <p>B. Measure and compare angles in degrees and radians.</p> <p>C. Determine relationships between linear, square, and cubic measures and describe how changes in one of the measures of the figure affect the others.</p> <p>D. Demonstrate ability to produce measures with specified levels of precision.</p>	<p>A. Select and use appropriate units and tools to collect and apply data with appropriate levels of precision.</p> <p>B. Apply degree and radian measure to solve real world problems.</p> <p>C. Describe the relationship between linear, square, and cubic measures and apply to real world problems.</p>
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2.4. Mathematical Reasoning and Connections

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<p>A. Construct algorithms for multi-step and real world problems.</p> <ul style="list-style-type: none"> • Describe connections between equivalent representations and corresponding procedures of the same problem situation or mathematical concept. <p>B. Constructs logical verifications or counter examples to test conjectures and to justify algorithms and solutions to</p>	<p>A. Use direct proofs, indirect proofs, or proof by contradiction to validate conjectures.</p> <p>B. Constructs algorithms for multi-step and real world problems.</p>	<p>A. Use direct proofs, indirect proofs, or proof by contradiction to validate conjectures.</p> <p>B. Construct valid arguments from stated facts.</p>	<p>B. Apply appropriate arguments, proof, and mathematical rules of logic in</p>

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<p>problems.</p> <p>D. Use if . . . then and converse statements to construct simple valid arguments.</p> <p>E. Solve equations that model problems in the sciences.</p>	<p>C. Relate inductive and deductive reasoning to make observations about and to verify properties of figures.</p> <ul style="list-style-type: none"> • Describe the difference between a statement that is verified by mathematical proof and one that is verified empirically using examples of data. <p>D. Use if . . . then, converse, inverse, and contra positive statements to construct simple valid arguments.</p>	<p>C. Determine the validity of an argument.</p> <p>D. Use truth tables to reveal the logic of mathematical statements.</p> <p>E. Demonstrate mathematical solutions to problems in the physical sciences.</p>	<p>problem solving situations.</p> <p>C. Create and write formal proofs of mathematical theorems including mathematical induction.</p> <p>E. Describe the application of graph theory to the study of networks.</p>

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2.5. Mathematical Problem Solving and Communication

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<p>A. Describe alternative methods or strategies to solve problems.</p> <p>B. Formulate and solve new problems based on existing examples.</p> <p>C. Create and write and solve problem situations and translate among oral, written, concrete, pictorial and graphical forms.</p>	<p>A. Analyze a variety of problem solving strategies to develop and defend more efficient strategies.</p> <p>B. Create and write problems extensions.</p> <p>C. Construct algorithms for solving multi-step and non-routine real world problems.</p>	<p>A. Select and use appropriate mathematical concepts and techniques from different areas of mathematics and apply them to solving non-routine and multi-step problems.</p> <p>B. Use symbols, mathematical terminology, standard notation, mathematical rules, graphing and other types of mathematical representations to communicate observations, predictions, concepts, procedures, generalizations, ideas and results.</p> <p>C. Present mathematical procedures and results clearly, systematically, succinctly and correctly.</p> <p>D. Conclude a solution process with a</p>	<p>A. Create and solve complex problems using appropriate mathematical concepts and techniques.</p> <p>B. Communicate both written and oral mathematical concepts and problems using appropriate mathematical language.</p> <p>C. Present mathematical procedures using necessary and sufficient language.</p>

D. Create and write a problem, determines information required to solve the problem, chooses methods for obtaining this information, and sets limits for acceptable solutions.	D. Explain the components of mathematical modeling: problem formulation, mathematical model, solution within the model, interpretation of solution within the model, validation in original real-world situation.	summary of results and evaluate the degree to which the results obtained represent an acceptable response to the initial problem and why the reasoning is valid.	D. Justify and defend solutions to mathematical and applied problems.
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2.6. Statistics and Data Analysis

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<p>C. Determine line of best fit for a scatter plot.</p> <ul style="list-style-type: none"> Approximate visually and calculate regression using graphics calculator. Determine strength of correlation for a scatter plot (e.g. strong, weak, or no correlation). Identify whether correlation is positive or negative (e.g. or shows growth or decay) 		<p>A. Design and conduct an experiment using random sampling, describe the data as an example of a distribution using statistical measures of center and spread, and organize and represent the results with graphs. (Use standard deviation, variance and t-tests.)</p> <p>B. Use appropriate technology to organize and analyze data taken from the local community.</p> <p>C. Determine regression equation of best fit (e.g., linear, quadratic, and exponential).</p> <p>D. Make predictions using interpolation,</p>	<p>A. Design, conduct, present, and defend a study that demonstrates an understanding of appropriate applied statistical methods (e.g. senior project statistics).</p>

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<ul style="list-style-type: none"> • <p>D. Make predictions using interpolation, extrapolation, regression, and estimation, using technology.</p> <p>E. Describe methods of data collection (e.g. Census, sample survey, observational study).</p> <ul style="list-style-type: none"> • Identify and describe sources of bias in surveys. (e.g., under coverage, voluntary response, wording of questions). • Identify types of samples (e.g. cluster, stratified). • Critique a survey using random sampling. <p>F. Display categorical data in two-way tables (e.g., classifying data according to categorical variables, i.e. male, female and freshman, sophomore)</p> <p>H. Calculate summary statistics of data distributions using mean, median, mode quartiles, range, and standard deviation.</p> <ul style="list-style-type: none"> • Compare and contrast different data distributions using summary statistics. 		<p>extrapolation, regression, and estimation, using technology.</p> <p>E. Determine the validity of the sampling method described in a given study.</p> <p>F. Determine the degree of dependence of two quantities specified by a two-way table.</p> <p>G. Describe questions of experimental design, use of control groups, treatment groups, cluster sampling and reliability.</p> <p>H. Use sampling techniques to draw inferences about large populations.</p>	

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<ul style="list-style-type: none"> • Determine the effect of changing units on summary measures. • Define standard deviation. • Select an appropriate data display for a data set and recognize when a graph does not give an accurate picture of a data set. • Explain the meaning of a confidence interval and margin or error. 		<p>I. Describe the normal curve and use its properties to answer questions about sets of data that are assumed to be normally distributed.</p>	

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2.7. Probability and Predictions

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<p>A. Define and calculate probability and odds of event.</p> <ul style="list-style-type: none"> • Describe and illustrate the “Law of Large Numbers” concept. 		<p>A. Compare odds and probability.</p> <p>B. Apply probability and statistics to perform an experiment involving a sample and generalize its results to the entire population.</p> <p>C. Draw a conclusion regarding the validity of a probability or statistical argument</p>	

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<p>E. Simulate a probability distribution of an event (e.g., tossing a coin, rolling dice, using technology)..</p> <ul style="list-style-type: none"> • Define and calculate conditional probability. • Define and distinguish between independent and dependent events. 	<p>D. Determine theoretical and experimental geometric probabilities using lengths and areas.</p>	<p>and justify conclusion.</p> <p>D. Use experimental and theoretical probability distributions to make judgments about the likelihood of various outcomes in uncertain situations.</p> <p>E. Solve problems involving independent simple and compound events.</p>	<p>D. Discuss the types of situations in which using simulations to estimate probability would be useful.</p> <ul style="list-style-type: none"> • Design and conduct simulation experiments and apply appropriate theoretical probability. • Conjecture about the difference between experimental and theoretical probability in real life situations. <p>E. Design, conduct, analyze, and present experiments that demonstrate an understanding of independent, mutually exclusive, union of events, and complimentary events.</p>

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2.8. Algebra and Functions

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<p>A. Simplify calculations, rewrite expressions and combine like terms using the distributive property.</p> <p>B. Determine the unknown values in patterns, sequences and series.</p> <p>C. Predict solutions to problems by using the graphs of equations (e.g., extrapolation, interpolation)</p> <p>D. Solve systems of equations by substitution and elimination.</p>	<p>A. Describe patterns using tables and variable expressions.</p> <p>B. Graph exponential and logarithmic functions using a graphing calculator.</p> <p>D. Solve systems of inequalities and matrices.</p> <p>E. Graph equations representing conic sections (e.g., circles, ellipses, parabolas and hyperbolas.)</p>	<p>A. Analyze a given set of data for the existence of a pattern and represent the pattern algebraically and graphically.</p> <p>B. Give examples of patterns that occur in data from other disciplines.</p> <p>C. Use patterns, sequences and series to solve routine and non-routine problems.</p> <p>D. Formulate expressions, equations, inequalities, systems of equations, systems of inequalities, and matrices to model routine and non-routine problem situations.</p> <p>E. Use equations to represent curves such as lines, circles, ellipses, parabolas, and hyperbolas.</p> <p>F. Identify whether systems of equations and inequalities are consistent or inconsistent.</p> <p>G. Analyze and explain systems of equations, systems of inequalities and matrices.</p> <p>H. Select and use an appropriate strategy to</p>	<p>A. Create mathematics models for geometric patterns and explain the reasoning used.</p> <p>B. Solve real life problems using patterns, sequences and series (e.g. interest, population growth).</p> <p>C. Collect, analyze, and model real life data that demonstrates and understanding of algebraic equations and systems of equations and inequalities.</p> <p>D. Apply systems of equations and inequalities to solve real life problems both graphically and algebraically.</p> <p>E. Apply linear programming strategies to solve business application problems.</p> <p>F. Solve real life applications of systems of equations and inequalities using graphing utilities.</p> <p>G. Create, write, and solve a matrix application problem.</p>

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<p>K. Select, justify and an appropriate technique to graph a linear function in two variables, including slope-intercept, x- and y- intercepts, graphing by transformations, and the use of a graphing calculator.</p> <p>L. Write the equations of a line when given the graph of the line, two points on the line, or the slope of the line and the point on the line.</p> <p>M. Given a set of data points, write an equation for a line of best fit.</p>	<p>P. Analyze a relation to determine whether</p>	<p>solve systems of equations and inequalities using graphing calculators, symbol manipulators, spreadsheets, and other software.</p> <p>I. Use matrices to organize and manipulate data, including matrix addition, subtraction, multiplication and scalar multiplication.</p> <p>J. Demonstrate the connection between algebraic equations and inequalities and the geometry of relations in the coordinate plane.</p> <p>K. Select, justify, and apply an appropriate technique to graph a linear function in two variables, including slope-intercept, x- and y-intercepts, graphing by transformations, and the use of a graphing calculator.</p> <p>L. Write the equation of a line when given the graph of the line, two points on the line, or the slope of the line and a point on the line.</p> <p>M. Given a set of data points, write an equation for a line of best fit.</p> <p>N. Solve linear, quadratic, and exponential equations both symbolically and graphically.</p> <p>O. Determine the domain and range of a relation, given a graph or set of ordered pairs.</p> <p>P. Analyze a relation to determine whether a direct or inverse variation exists and represent it algebraically and graphically.</p> <p>Q. Represent functional relationships in</p>	<p>H. Describe the connection between the binomial expansion and Pascal's triangle and apply to permutations and combinations problems.</p> <p>I. Collect and model real life data using polynomial functions, exponential and power functions.</p> <p>J. Solve problems involving direct, inverse and joint variation.</p> <p>K. Create, write, and solve real life problems that demonstrate an understanding of appropriate function models.</p>

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	<p>a direct or inverse variation exists and represent it algebraically and graphically.</p>	<p>tables, charts, and graphs.</p> <p>R. Create and interpret functional models.</p> <p>S. Analyze properties and relationships of functions (linear, polynomial, rational, trigonometric, exponential, and logarithmic).</p> <p>T. Analyze and categorize functions by their characteristics.</p>	

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2.9. Geometry

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<p>A. Use a graphics calculator to model algebraic functions on a coordinate plane.</p> <p>B. Construct deductive proofs involving a coordinate system (e.g. use slope to prove that a quadrilateral is a rectangle.)</p> <p>C. Calculate the areas of parallelograms, triangles, rhombi, trapezoids, regular polygons, and circles. Identify and name faces, edges and vertices of polyhedron. Describe and define properties of geometric solids.</p>	<p>A. Construct geometric figures using dynamic geometry tools (Geometer's Sketchpad, Cabri Geometre).</p> <ul style="list-style-type: none"> • Identify and draw medians, altitudes, angle bisectors, and perpendicular bisectors in a triangle. <p>B. Prove two triangles or two polygons are congruent or similar using algebraic and coordinate as well as deductive proofs.</p> <p>C. Identify and prove the properties of quadrilaterals involving opposite sides and angles, consecutive sides and angles, and diagonals using deductive proofs.</p> <ul style="list-style-type: none"> • Apply properties of parallel lines to determine angle measures. 	<p>A. Construct geometric figures using dynamic geometry tools (Geometer's Sketchpad, Cabri Geometre).</p> <p>B. Prove two triangles or two polygons are congruent or similar using algebraic and coordinate as well as deductive proofs.</p> <p>C. Identify and prove the properties of quadrilaterals involving opposite sides and angles, consecutive sides and angles, and diagonals using deductive proofs.</p>	<p>A. Describe (e.g. center, vertices, foci, asymptotes) conic sections from general equations.</p>

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<p>D. Solve problems involving the proportional relationship of corresponding parts of similar figures (polygons).</p> <p>G. Apply the properties of parallel and perpendicular lines, slope, distance, midpoint on a coordinate system to classify polygons.</p> <p>H. Translate, reflect, rotate and dilate geometric shapes on a coordinate system.</p> <p>I. Use geometric models to solve equations (e.g., find x given a relationship in a pair of complementary or</p>	<p>D. Identify corresponding parts in congruent triangles to solve problems.</p> <ul style="list-style-type: none"> Describe relationships between sides and angles in a triangle. Create and identify congruent polygons and their corresponding parts. <p>E. Solve problems involving inscribed and circumscribed polygons.</p> <p>F. Use the properties of angles, arcs, chords, tangents, and secants to solve problems involving circles.</p> <ul style="list-style-type: none"> State the relationships between arcs, chords, diameters, angles, and tangents of circles. <p>G. Solve problems using analytic geometry.</p> <p>H. Construct a geometric figure and its image using various transformations.</p> <p>I. Model situations geometrically to formulate and solve problems.</p> <ul style="list-style-type: none"> Apply angle measures of polygons to solve problems. 	<p>D. Identify corresponding parts in congruent triangles to solve problems.</p> <p>E. Solve problems involving inscribed and circumscribed polygons.</p> <p>F. Use the properties of angles, arcs, chords, tangents, and secants to solve problems involving circles.</p> <p>G. Solve problems using analytic geometry.</p> <p>H. Construct a geometric figure and its image using various transformations.</p> <p>I. Model situations geometrically to formulate and solve problems.</p>	<p>F. Demonstrate appropriate use of vectors and vector operations to solve real world applications (bearings).</p>

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<p>supplementary angles).</p> <p>J. Justify symmetry of shapes on a coordinate system (e.g., show a triangle is isosceles using a line of symmetry).</p>	<ul style="list-style-type: none"> • Solve problems involving area and geometric probability. • Determine the surface area and volume of prisms, cylinders, pyramids, cones, and spheres. • Perform operations with vectors and determine the direction and magnitude of a vector. <p>J. Analyze figures in terms of the kinds of symmetries they have.</p>	<p>J. Analyze figures in terms of the kinds of symmetries they have.</p>	

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2.10. Trigonometry

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		<p>A. Use graphing calculators to display periodic and circular functions; describe properties of the graphs.</p>	<p>A. Describe, mathematically and verbally, classify, and apply periodic and circular function.</p> <ul style="list-style-type: none"> • Collect appropriate data, analyze period, amplitude and phase shifts, and describe model for applications of sine and cosine functions. (e.g.

	<p>B. Express trigonometric ratios as fractions or decimals and describe their relationships to right triangles.</p> <ul style="list-style-type: none"> • Apply the properties of special (45-45-90 and 30-60-90) right triangles to real life situations. • Calculate values of trigonometric ratios related to the measure of an angle. 	<p>B. Identify, create, and solve practical problems involving right triangles using the trigonometric functions and the Pythagorean Theorem.</p>	<p>harmonic motion).</p> <ul style="list-style-type: none"> • Create write and solve real world problems that demonstrate an understanding of inverse trigonometric functions. <p>B. Create, write, and solve real word application problem that demonstrate understanding of solving right triangles and/or using the law of sines and/or law of cosines.</p> <ul style="list-style-type: none"> • Represent and apply translations of sine and cosine functions. • Use the law of sines and the law of cosines to determine sides and/or angles of a triangle. • Prove equivalent trigonometric expressions.
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2.11. Concepts of Calculus

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<p>A. Indicate whether a given relation is also a function given a set of ordered pairs.</p> <ul style="list-style-type: none"> • Identify intervals over which a function increases, decreases, or remains constant using a graphing calculator (graphically). • Compute the domain and range of a function from its equation. • Define the concept of continuity. • Locate discontinuities of functions graphically. 		<p>A. Determine maximum and minimum values of a function over a specified interval.</p>	

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<p>C. Relate and interpret slope of a line as the variation constant in a direct variation.</p>		<p>B. Interpret maximum and minimum values in problem situations.</p> <p>C. Graph and interpret rates of growth/decay.</p> <p>D. Determine sums of finite sequences of numbers and infinite geometric series.</p> <p>E. Estimate areas under curves using sequences of areas.</p>	<p>B. Describe the meaning of maximum or minimum values of a function and how it applies to a real life situation.</p> <p>C. Collect real world data that represents growth/decay, describe the mathematical model of the graph of the data, and make predictions based on the model.</p> <p>D. Create, write, and solve real world application problems that demonstrate an understanding of arithmetic and geometric sequences and series.</p> <p>E. Describe the method for estimating the area under curves and apply to real world situations.</p>

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